Analytical Studies on Static Aeroelastic Behavior of Forward-Swept Composite Wing Structures

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An exact methodology allowing one to determine the aeroelastic lift distribution and the divergence instability of swept cantilevered composite wing structures is developed in this paper. The approach based on the Laplace transform technique enables one to solve, in a unified manner, both aeroelastic problems. The analysis encompasses the cases of free and constrained warping models for the wing twist. Numerical results are presented to demonstrate the effects played by the fiber orientation, ply lay-up, warping inhibition, and wing geometry on the subcritical static aeroelastic response and on the divergence instability of composite swept wings.

Nomenclature

a_0	= two-dimensional lift curve slope coefficient
$\mathcal{R}$	= wing aspect ratio
B_{ij}	= bending-stretching rigidities
c	= wing chord measured perpendicular to the reference axis
D_{ij}	= bending rigidities of the composite wing
$\tilde{D}_{ij}$	= composite coupling stiffness rigidities
d	= the distance between the line of center of mass and the elastic axis
e	= distance between the spanwise reference axis and aerodynamic centerline
G	= composite nondimensional torsional coupling parameter
K	= composite nondimensional bending coupling parameter
L	= aerodynamic lift force
l	= wing semispan measured along the reference axis
N	= load factor normal to the wing surface
q	= dynamic pressure, $\equiv \frac{1}{2}\rho V^2$
S	= warping rigidity (nondimensional)
s	= Laplace transform variable
T	= aerodynamic torsional moment
V	= freestream velocity (parallel to the longitudinal axis of the airplane)
y	= spanwise coordinate
$\tilde{Z}$	= bending deflection of the reference axis (nondimensional), $\equiv Z/l$
$\tilde{Z}_0^{(r)}$	$= d^r\tilde{Z}/d\eta^r\|_{\eta=0}$
α	= torsional deflection of wing sections (positive nose up)
α_0	= rigid wing angle of attack
$\alpha_0^{(r)}$	$= d^r\alpha/d\eta^r\|_{\eta=0}$
η	$= y/l$
$\theta_{(j)}$	= orthotropicity angle of each lamina j with respect to a rearward normal to the reference axis (positive for swept back)
Λ	= sweep angle of the reference axis (positive for swept back)

Subscripts

n	= component normal to the leading edge
$(\ )_{,11}$	$= d^2(\ )/d\eta^2$

Superscript

$'$	$= d(\ )/dy$

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Introduction

A GREAT deal of interest for the utilization of laminated composite structural systems in the aeronautical and aerospace industries has emerged in recent years. This interest was stimulated by the development of high-modulus, high-strength, low-weight composite materials as well as by the possibility to use adequately their exotic characteristics (directionality and ply stacking sequence) in order to synthesize a structure with enhanced aeroelastic performance. Employed in an earlier stage in the supersonic panel flutter problems (see, e.g., Ref. 1), the aeroelastic tailoring concept was applied later to swept-forward wing aircraft. As is well known, the application of this new technology has resulted in the possibility of practically eliminating (without weight penalties) the occurrence of the aeroelastic divergence instability (a chronic problem facing the free use of this type of airplane). Most recently, it has culminated successfully with the construction of the Grumman X29 research airplane.

The tailoring concept applied to composite lifting surfaces, in general, and to forward-swept wings (FSW), in particular, has been discussed thoroughly in a series of recent survey papers. In this sense, the reader is referred to Refs. 2–5 where ample references to the available research works are given. Along with the great, undoubted advantages conferred by the employment of structural composites, a series of challenges are associated with the use of these composite materials.

Part of these challenges are due to the complexities arising from the anisotropic nature of composite materials themselves and the multitude of structural couplings, not extant in the case of traditional metallic structures. In addition, as the results concerning the structural[6-10] and aeroelastic[11-16] behavior of cantilevered structures fully reveal, in the case of composite materials the effects played by the axial warping restraint become more prominent and more complex than in the case of their metallic counterparts.[17,18] Moreover, incorporation of this effect yields an increase of the order of aeroelastic equilibrium equations, which, in its turn, constitutes a supplementary mathematical complexity toward the determination of the solution of aeroelastic problems. As is well known, the exact solution to the differential equations for divergence and lift distribution as developed by Diederich and Budiansky[19] for metallic swept wings [obtained within the free warping

(FW) model] has been extended to swept-forward composite wings by Weisshaar.[20,21] However, in spite of its mathematical elegance, this solution is not versatile enough to accommodate the incorporation of the warping restraint effect. Within the framework of this paper, an exact solution technique is presented allowing one to study, in a unified manner, both the aeroelastic lift distribution and the divergence instability of FSW composite wing structures by incorporating the warping restraint effect. Needless to say, these static problems are of an increasing significance in the design of modern high-performance aircraft. Although the method is illustrated here for the static case, it could be extended to study the subcritical dynamic response and the flutter instability as well. This paper could be viewed as a generalization and unification of the previous papers of the authors.[14-16] Within this paper, the wing structure will be considered restrained against rigid-body motion. Studies devoted to the aeroelastic divergence of unrestrained vehicles can be found, for example, in Refs. 22-30.

Basic Assumptions, Aeroelastic Governing Equations

A simple anisotropic plate-beam model assumed to have all its bending and torsional stiffness derived from the laminated composite upper and lower skins is used to analyze the aeroelastic phenomena mentioned earlier. This model is based on the kinematical assumptions used previously by Housner and Stein[31] and Weisshaar.[20] As in Refs. 20, 21, and 32, we shall postulate the existence of a reference axis (RA) coinciding with the y axis and located, for the sake of simplicity, at the midchord of the wing.

Consistent with the assumptions formulated in Ref. 12 (for the geometrical characteristics of the wing see Fig. 1 as well as the definitions given in the Nomenclature), the equations governing the static aeroelastic equilibrium of uniform cross-section composite swept wings written in the form deduced in Ref. 12 are

$$\tilde{D}_{22}Z'''' - \tilde{D}_{66}\alpha''' = L(y)$$

$$(c^3/12)\, D_{22}\alpha'''' - \tilde{D}_{66}\alpha'' + \tilde{D}_{26}Z''' = T(y) \tag{1}$$

In these equations, $\tilde{D}_{22}$, $\tilde{D}_{26}$, and $\tilde{D}_{66}$ denote the coupling rigidities of the composite anisotropic wing (defined in the Appendix); D_{22} and $c^3D_{22}/12$ denote the bending and the warping rigidity, respectively; $Z(y)$ and $\alpha(y)$ denote the deflection of the reference axis and the twist around this axis, respectively; and $L(y)$ and $T(y)$ represent the lift and the

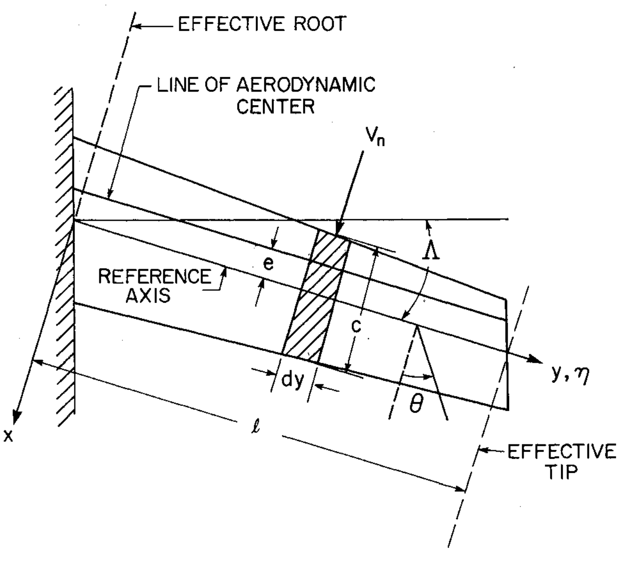

Fig. 1 Geometry of the swept-wing aircraft.

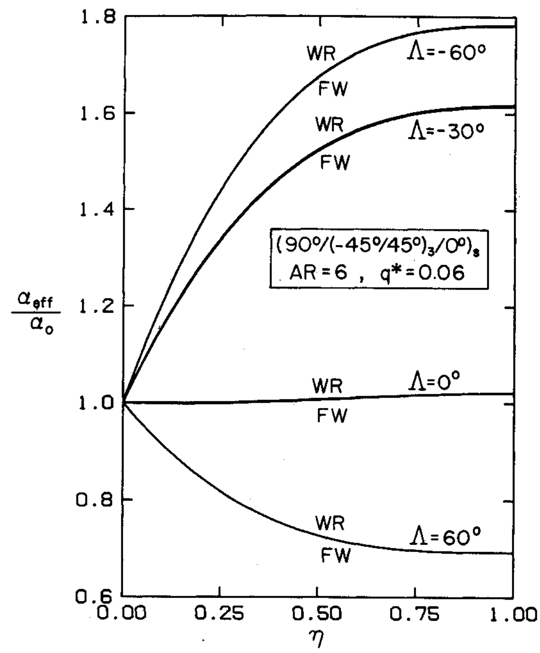

Fig. 2 Normalized elastic lift distribution on a uniform composite swept wing (structure a, Æ = 6).

aerodynamic nose-up torsional moment (per unit span), respectively. For the static case considered here, $L(y)$ and $T(y)$ are[32]

$$L(y) = a_0 q_n c(\alpha_0 + \alpha - Z' \tan\Lambda) - NW/2l$$

$$T(y) = a_0 q_n ce(\alpha_0 + \alpha - Z' \tan\Lambda)$$

$$-NWd/2l + q_n c^2 C_{mac} \tag{2}$$

where q_n ($\equiv \rho_0 V_n^2/2$) $= q\cos^2\Lambda$ denotes the dynamic pressure component normal to the leading edge; a_0 [$\equiv 2\pi Æ/(Æ + 4\cos\Lambda)$] denotes the approximation for three-dimensional slope of the lift curve valid for subsonic and subcritical Mach numbers[19,33]; α_0 denotes the rigid wing angle of attack (measured in planes normal to the leading edge); C_{mac} is the wing section pitching moment coefficient; and $W/2l$ is the wing weight per unit span. The boundary conditions (BCs) associated with these equations are[12]

At $y = 0$:

$$Z = Z' = \alpha = \alpha' = 0$$

At $y = l$:

$$\tilde{D}_{22}Z'' - \tilde{D}_{26}\alpha' = 0$$

$$\tilde{D}_{22}Z''' - \tilde{D}_{26}\alpha'' = 0$$

$$-c^3D_{22}/12\alpha''' + \tilde{D}_{66}\alpha' - \tilde{D}_{26}Z'' = 0$$

$$\alpha'' = 0 \tag{3}$$

In the case of the free warping model, the aeroelastic equilibrium equations are[12,20,21]

$$\tilde{D}_{22}Z'''' - \tilde{D}_{26}\alpha''' = L(y)$$

$$\tilde{D}_{66}\alpha'' - \tilde{D}_{26}Z''' = -T(y) \tag{4}$$

while the associated BCs become

At $y = 0$:

$$Z = Z' = \alpha = 0$$

At $y = l$:

$$\tilde{D}_{22}Z'' - \tilde{D}_{26}\alpha' = 0$$

$$\tilde{D}_{22}Z''' - \tilde{D}_{26}\alpha'' = 0$$

$$\tilde{D}_{66}\alpha' - \tilde{D}_{26}Z'' = 0 \tag{5}$$

Nondimensional Form of Aeroelastic Governing Equations

Equations (1) and (4) and their associated BCs [Eqs. (3) and (5), respectively] will be converted to a nondimensional form. To achieve this goal, the following nondimensional parameters are defined: $\tilde{Z}(\equiv Z/l)$ and $\eta(\equiv y/l)$, denoting the nondimensional deflection and the spanwise coordinate, respectively; $K(\equiv \tilde{D}_{26}/\tilde{D}_{22})$ and $G(\equiv \tilde{D}_{26}/\tilde{D}_{66})$, the nondimensional bending and torsional coupling stiffnesses, respectively; $S[\equiv c^3 D_{22}/(12 l^2 \tilde{D}_{66})]$, denoting the nondimensional warping rigidity;

$$a_1 \equiv q_n c l^3 a_0/\tilde{D}_{22}$$

$$a_2 \equiv q_n e c l^2 a_0/\tilde{D}_{66} \tag{6}$$

denote two speed parameters associated with the bending and torsional degrees of freedom, respectively, connected by $a_2 = \kappa a_1$ where $\kappa \equiv eG/lK$. Converted to the nondimensional form, the equations of static aeroelastic equilibrium for the case where warping restraint effect is incorporated are

$$\tilde{Z}_{,1111} - K\alpha_{,111} - a_1(\alpha - \tilde{Z}_{,1}\tan\Lambda) = a_1\alpha_0 - NWl^2/2\tilde{D}_{22}$$

$$S\alpha_{,1111} - \alpha_{,11} + G\tilde{Z}_{,111} - a_2(\alpha - \tilde{Z}_{,1}\tan\Lambda)$$

$$= a_2(\alpha_0 + cC_{mac}/e) - NWdl/2\tilde{D}_{66} \tag{7}$$

while their associated BCs are given by

At $\eta = 0$:

$$\tilde{Z} = \tilde{Z}_{,1} = \alpha = \alpha_{,1} = 0$$

At $\eta = 1$:

$$\tilde{Z}_{,11} - K\alpha_{,1} = 0$$

$$\tilde{Z}_{,111} - K\alpha_{,11} = 0$$

$$-S\alpha_{,111} + \alpha_{,1} - G\tilde{Z}_{,11} = 0$$

$$\alpha_{,11} = 0 \tag{8}$$

For the free warping case, the equations become

$$\tilde{Z}_{,1111} - K\alpha_{,111} - a_1(\alpha - \tilde{Z}_{,1}\tan\Lambda) = a_1\alpha_0 - NWl^2/2\tilde{D}_{22}$$

$$G\tilde{Z}_{,111} - \alpha_{,11} - a_2(\alpha - \tilde{Z}_{,1}\tan\Lambda) = a_2(\alpha_0 + cC_{mac}/e)$$

$$-NWdl/2\tilde{D}_{66} \tag{9}$$

while the associated BCs are

At $\eta = 0$:

$$\tilde{Z} = \tilde{Z}_{,1} = \alpha = 0$$

At $\eta = 1$:

$$\tilde{Z}_{,11} - K\alpha_{,1} = 0$$

$$\tilde{Z}_{,111} - K\alpha_{,11} = 0$$

$$\alpha_{,1} - G\tilde{Z}_{,11} = 0 \tag{10}$$

In Eqs. (7–10), $()_{,1} \equiv d()/d\eta$. It should be noted that the load factor N normal to the elastic swept wing is expressible as

$$N = \frac{2clq_n a_0}{W}\int_0^1 (\alpha_0 + \alpha - \tilde{Z}_{,1}\tan\Lambda)d\eta \tag{11}$$

Static Aeroelastic Response

To determine the static response characteristics of the composite wing structure [i.e., $\tilde{Z} = \tilde{Z}(\eta)$ and $\alpha \equiv \alpha(\eta)$], Eqs. (7) (obtained within the warping model) have to be solved in conjunction with Eq. (11) and the associated boundary conditions [Eqs. (8) and (10), respectively]. This will be done through the application of the Laplace transform technique. As it may be inferred, Eqs. (7) and (9) considered in conjunction with Eq. (11) constitute an integral-differential equation system. However, in order to reduce the solution of the problem to that of a differential equation system, an iteration process has to be used. Consistent with this procedure, the load factor N in Eqs. (7) and (9) is assimilated to a constant that has to be re-evaluated at each step of the iterative procedure. Alternative methods for treating this problem, applied to free warping composite wings, can be found in Refs. 20 and 34.

Solution Procedure

The Laplace integral transform method is used to solve exactly the equations governing the static aeroelastic response of swept composite wing structures. From the mathematical point of view, the problems associated with warping restraint and free warping models are similar. That is why in the following only the solution procedure associated with the equations incorporating warping restraint effect will be outlined. The

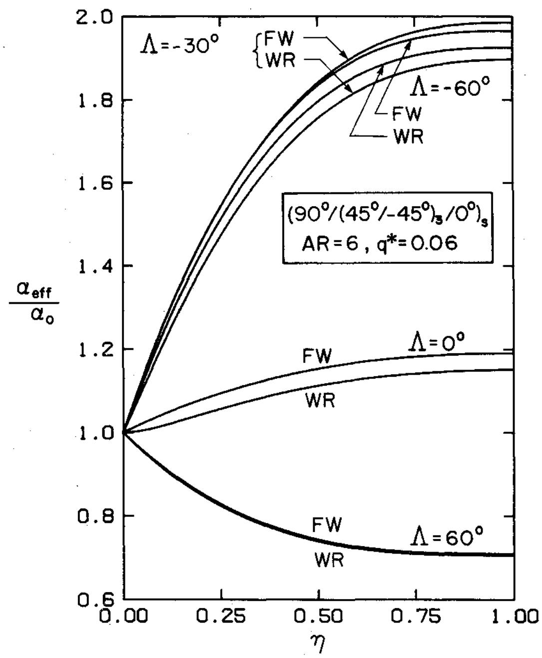

Fig. 3 Normalized elastic lift distribution on a uniform composite swept wing (structure b, Æ = 6).

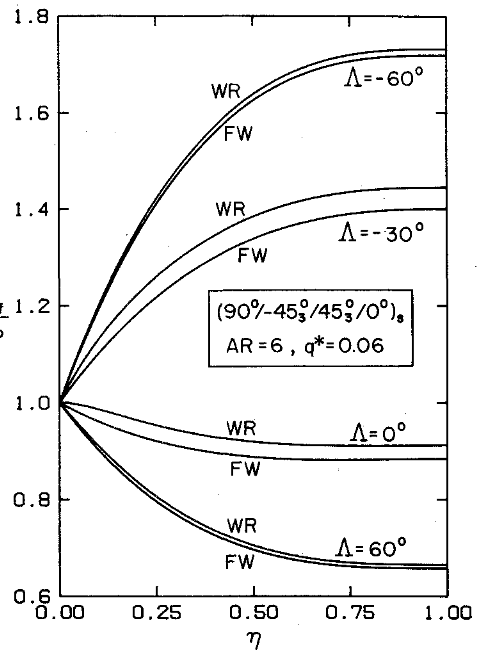

Fig. 4 Normalized elastic lift distribution on a uniform composite swept wing (structure c, $\mathcal{R}$ = 6).

Laplace transform (LT) of the nth-order derivative of the generic function $f \equiv f(\eta)$ is

$$L\left[\frac{d^n f(\eta)}{d\eta^n}\right] = s^n F(s) - s^{n-1} f\Big|_{\eta=0} - s^{n-2}\left(\frac{df}{d\eta}\right)\Big|_{\eta=0}$$

$$- \cdots - \left(\frac{d^{n-1}f}{d\eta^{n-1}}\right)\Big|_{\eta=0} \tag{12}$$

where $F(s) \equiv L[f(\eta)]$; L denotes the Laplace transform operator. From Eqs. (7), one obtains the solution in the LT space as

$$L[\tilde{Z}(\eta)] = \frac{P_1(s)}{g(s)} + \frac{P_2(s)}{sg(s)} + \frac{P_3(s)}{sg(s)}$$

$$L[\alpha(\eta)] = \frac{R_1(s)}{g(s)} + \frac{R_2(s)}{sg(s)} + \frac{R_3(s)}{sg(s)} \tag{13}$$

where $g(s)$ is expressible as

$$g(s) = s^3 Q(s) \tag{14}$$

In Eqs. (13) and (14), $P_i(s)$, $R_i(s)$ $(i = \overline{1,3})$, and $Q(s)$ are polynomials not displayed here. The polynomials Q, P_1, and R_1 depend on the characteristics of the aeroelastic flow-structure system, whereas the last two also depend on the boundary conditions at the root section; the polynomials P_i and R_i $(i = 2,3)$ are also correlated with the characteristics of the aeroelastic flow structure and the load factor. Toward the end of inverting the solution into the physical space, the partial fraction expansion will be used. To this end, the fractions occuring in Eqs. (13) are recast conveniently as

$$L[\tilde{Z}(\eta)] = \frac{\overset{(I)}{P_1}(s)}{s^3} + \frac{\overset{(II)}{P_1}(s)}{Q(s)} + \frac{\overset{(I)}{P_2}(s)}{s^4} + \frac{\overset{(II)}{P_2}(s)}{Q(s)} + \frac{\overset{(I)}{P_3}(s)}{s^4} + \frac{\overset{(II)}{P_3}(s)}{Q(s)}$$

$$L[\alpha(\eta)] = \frac{\overset{(I)}{R_1}(s)}{s^3} + \frac{\overset{(II)}{R_1}(s)}{Q(s)} + \frac{\overset{(I)}{R_2}(s)}{s^4} + \frac{\overset{(II)}{R_2}(s)}{Q(s)} + \frac{\overset{(I)}{R_3}(s)}{s^4} + \frac{\overset{(II)}{R_3}(s)}{Q(s)} \tag{15}$$

Here the polynomials occuring in Eqs. (15) may be obtained easily from the ones in Eqs. (13). Observing that the polynomial $Q(s)$ has five distinct roots λ_i $(i = \overline{1,5})$, the solution for $Z(\eta)$ may be be expressed formally as

$$Z(\eta) = \frac{1}{2}\lim_{s\to 0}\frac{d^2}{ds^2}\left[\overset{(I)}{P_1}(s)e^{s\eta}(I)\right] + \frac{1}{3!}\lim_{s\to 0}\frac{d^3}{ds^3}\left[\overset{(II)}{P_2}(s)e^{s\eta}\right]$$

$$+ \frac{1}{3!}\lim_{s\to 0}\frac{d^3}{ds^3}\left[\overset{(I)}{P_3}(s)e^{s\eta}\right] + \sum_{i=1}^{5}\frac{\overset{(II)}{P_1}(\lambda_i)}{(dQ/ds)|_{s=\lambda_i}}e^{\lambda_i\eta}$$

$$+ \sum_{i=1}^{5}\frac{\overset{(II)}{P_2}(\lambda_i)}{(dQ/ds)|_{s=\lambda_i}}e^{\lambda_i\eta} + \sum_{i=1}^{5}\frac{\overset{(II)}{P_3}(\lambda_i)}{(dQ/ds)|_{s=\lambda_i}}e^{\lambda_i\eta} \tag{16}$$

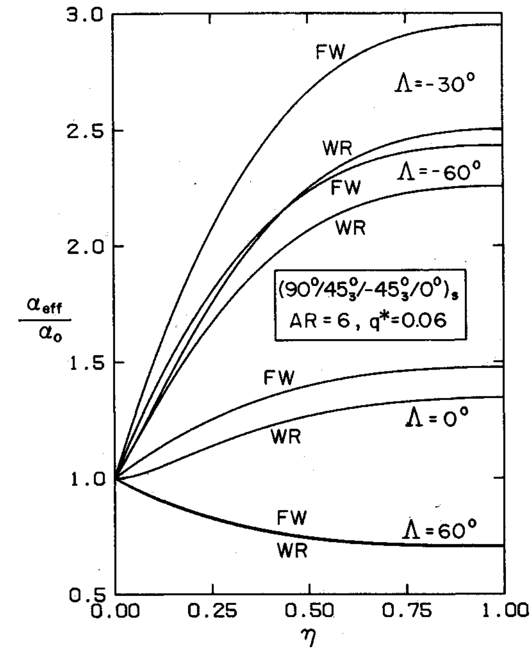

Fig. 5 Normalized elastic lift distribution on a uniform composite swept wing (structure d, $\mathcal{R}$ = 6).

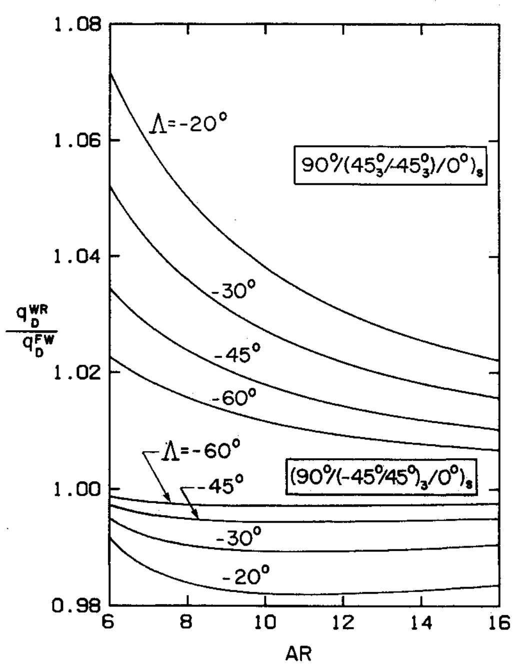

Fig. 6 Effect played by the warping inhibition on the divergence of swept-forward composite wings for structures b and a (the upper and bottom halves of the figure, respectively).

whereas for $\alpha(\eta)$, a similar expression as for $\tilde{Z}(\eta)$ could be obtained. By ordering and collecting terms in Eqs. (15) and (16), one may express the solution in the form

$$\begin{aligned}\tilde{Z}(\eta) &= \tilde{Z}_0^{(2)}F_1(\eta) + \tilde{Z}_0^{(3)}F_2(\eta) + \alpha_0^{(2)}F_3(\eta) \\ &+ \alpha_0^{(3)}F_4(\eta) + \mathfrak{F}_1(\eta) \\ \alpha(\eta) &= \tilde{Z}_0^{(2)}F_5(\eta) + \tilde{Z}_0^{(3)}F_6(\eta) + \alpha_0^{(2)}F_7(\eta) \\ &+ \alpha_0^{(3)}F_8(\eta) + \mathfrak{F}_2(\eta)\end{aligned} \tag{17}$$

Here,

$$\tilde{Z}_0^{(r)} \equiv \left.\frac{d^r\tilde{Z}(\eta)}{d\eta^r}\right|_{\eta=0} \tag{18}$$

and

$$F_i(\eta) = \sum_{r=1}^{5} \Omega_{ir}\exp(\lambda_r\eta), \qquad (i = \overline{1,8})$$

play the role of shape functions where Ω_{ir} depend on the characteristics of the flow-structure system, whereas $\mathfrak{F}_i$ $(i = 1,2)$ are cubic polynomials in η whose coefficients depend on the aerodynamic characteristics of the rigid wing counterpart and on the load factor N. This system of equations is basic in deriving both the subcritical aeroelastic response and divergence instability characteristics of wing structures incorporating the warping inhibition effect. Enforcement, in conjunction with Eqs. (8) of BCs at the wingtip ($\eta = 1$), results in a system of four nonhomogeneous equations written in matrix form as

$$\begin{bmatrix}\xi_{11} & \cdots & \xi_{14}\\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \xi_{41} & \cdots & \xi_{44}\end{bmatrix}\begin{Bmatrix}\tilde{Z}_0^{(2)}\\ \tilde{Z}_0^{(3)}\\ \alpha_0^{(2)}\\ \alpha_0^{(3)}\end{Bmatrix} = \begin{Bmatrix}f_1\\ f_2\\ f_3\\ f_4\end{Bmatrix} \tag{19}$$

where ξ_{ij} $(i,j = \overline{1,4})$ are constant-valued quantities depending on the flow-structure system characteristics, whereas the constants f_i $(i = \overline{1,4})$ are correlated with the aerodynamic charac-

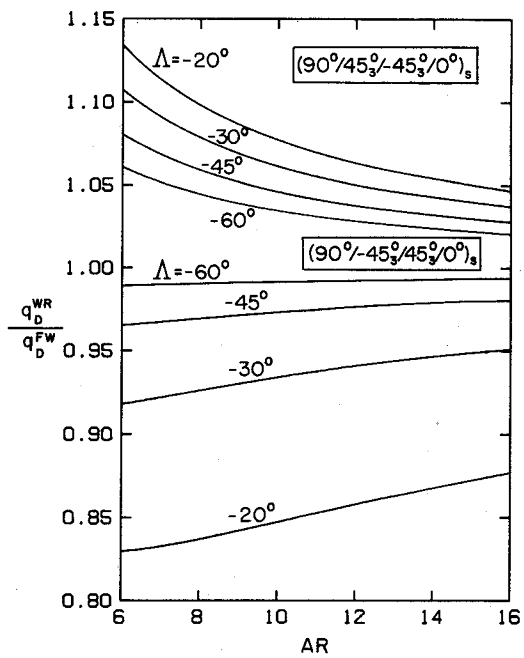

Fig. 7 Effect played by the warping inhibition on the divergence of swept-forward composite wings for structures d and c (the upper and bottom halves of the figure, respectively).

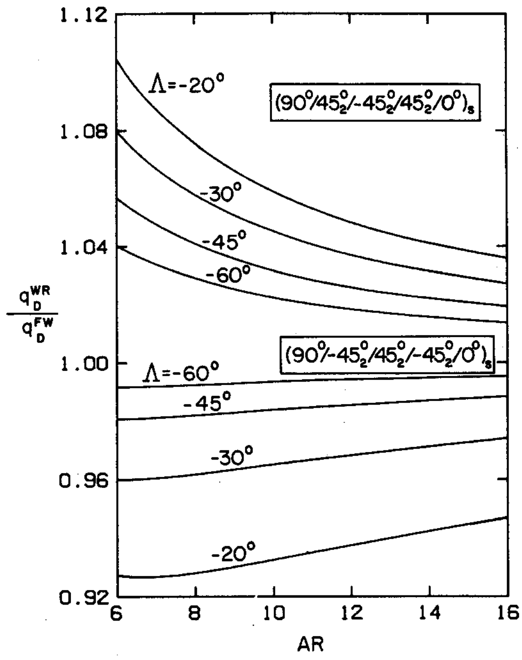

Fig. 8 Effect played by the warping inhibition on the divergence of swept-forward composite wings for structures f and e (the upper and bottom halves of the figure, respectively).

teristics of the rigid wing and the load factor N. The solution to the nonhomogeneous equation system [Eq. (19)] yields the vector

$$\{\tilde{Z}_0^{(2)}, \tilde{Z}_0^{(3)}, \alpha_0^{(2)}, \alpha_0^{(3)}\}^T$$

and as a result, from Eqs. (17), we may fully determine $\tilde{Z}(\eta)$ and $\alpha(\eta)$. These quantities allow further determination of the effective local angle of attack α_{eff}:

$$\alpha_{\text{eff}} = \alpha_0 + \alpha - \tilde{Z}_{,1}\tan\Lambda \tag{20}$$

The effect on the lift distribution due to elasticity may be illustrated in terms of the ratio $\alpha_{\text{eff}}/\alpha_0$:

$$\frac{\alpha_{\text{eff}}}{\alpha_0}\left(\equiv\frac{C_{L_e}}{C_{L_r}}\right) = 1 + \frac{\alpha - \tilde{Z}_{,1}\tan\Lambda}{\alpha_0} \tag{21}$$

In addition to Eq. (21), we may determine the ratios L_e/L_r and M_e/M_r where L_e (L_r) and M_e (M_r) denote the net lift force and the aerodynamic bending moment on the elastic (and on its rigid wing counterpart), respectively, where the bending moments are determined about the effective root of the wing.

Determination of the Divergence Instability

For a divergence instability analysis, Eq. (19) should be considered as homogeneous. In this case, the argument of nontriviality of the solution in Eq. (19) [i.e., of $\tilde{Z}(\eta)$ and $\alpha(\eta)$] results in the condition of divergence expressed in determinantal form. The divergence condition is thus obtained by vanishing the determinant of the matrix ξ_{ij} $(i,j = \overline{1,4})$. The minimum positive root a_1 of the obtained equation determines the divergence instability speed $(a_1)_D$.

Free Warping Model

In this case, by paralleling the procedure already outlined, one may express the solution in the form

$$\tilde{Z}(\eta) = \tilde{Z}_0^{(2)}\hat{F}_1(\eta) + \alpha_0^{(1)}\hat{F}_2(\eta) + \alpha_0^{(2)}\hat{F}_3(\eta) + \hat{\mathfrak{F}}_1(\eta)$$

$$\alpha(\eta) = \tilde{Z}_0^{(2)}\hat{F}_4(\eta) + \alpha_0^{(1)}\hat{F}_5(\eta) + \alpha_0^{(2)}\hat{F}_6(\eta) + \hat{\mathfrak{F}}_2(\eta) \tag{22}$$

where the polynomials $\tilde{\mathfrak{F}}_i(\eta)$ and $\hat{\mathfrak{F}}_i(\eta)$ have the same meaning as their counterparts in Eqs. (17). As in the case of the warping restraint model, a similar procedure yields a system of three nonhomogeneous equations:

$$\begin{bmatrix} \hat{\xi}_{11} & \cdots & \hat{\xi}_{13} \\ \cdots & \cdots & \cdots \\ \hat{\xi}_{31} & \cdots & \hat{\xi}_{33} \end{bmatrix} \begin{Bmatrix} \tilde{Z}_0^{(2)} \\ \alpha_0^{(1)} \\ \alpha_0^{(2)} \end{Bmatrix} = \begin{Bmatrix} \hat{f}_1 \\ \hat{f}_2 \\ \hat{f}_3 \end{Bmatrix} \tag{23}$$

which represents the free warping counterpart of Eq. (19).

Numerical Results

Two static aeroelastic problems playing a great importance in the design of composite swept lifting surfaces and implicitly of modern high-performance aircraft (see, e.g., Ref. 35) have been considered in this paper. Their response behavior from this point of view is investigated according to the procedure outlined in the preceding sections.

A symmetrical 16-layer composite wing structure considered in six variants will be used to obtain numerical results. Throughout these cases, a single material, namely, boron-epoxy, is used as a constituent of the layers. Its mechanical constants are

$$E_1 = 30.0 \times 10^6 \text{ psi}, \qquad E_2 = 3.0 \times 10^6 \text{ psi}$$

$$G_{12} = 1.0 \times 10^6 \text{ psi}, \qquad \nu_{12} = 0.30$$

These six arrangement variants are defined and labeled as

Structure a: $[90 \text{ deg}/(-45 \text{ deg}/45 \text{ deg})_3/0 \text{ deg}]_s$
Structure b: $[90 \text{ deg}/(45 \text{ deg}/-45 \text{ deg})_3/0 \text{ deg}]_s$
Structure c: $(90 \text{ deg}/-45 \text{ deg}_3/45 \text{ deg}_3/0 \text{ deg})_s$
Structure d: $(90 \text{ deg}/45 \text{ deg}_3/-45 \text{ deg}_3/0 \text{ deg})_s$
Structure e: $(90 \text{ deg}/-45 \text{ deg}_2/45 \text{ deg}_2/-45 \text{ deg}_2/0 \text{ deg})_s$
Structure f: $(90 \text{ deg}/45 \text{ deg}_2/-45 \text{ deg}_2/45 \text{ deg}_2/0 \text{ deg})_s$

Throughout these calculations, it was considered $e/c = 0.15$ and $\alpha_0 = 5$ deg. Figures 2–5 display the distribution of the normalized effective angle of attack $\alpha_{\text{eff}}/\alpha_0$ vs the nondimensional spanwise coordinate η, whereas Figs. 6–8 display, for several of these structures, the variation of the ratio q_D^{WR}/q_D^{FW} (i.e., of the ratio of the divergence dynamic pressure for the wing incorporating the warping inhibition to its counterpart obtained within the free warping model) vs the wing aspect ratio Æ.

The values for the dynamic pressure parameter $q^*(\equiv ql^3/E_1h^3)$, sweep angle Λ, and aspect ratio Æ are specified on the respective graphs.

Conclusions

The structural wing configurations resulting in those curves (see Figs. 2–5) falling below and above the line $\alpha_{\text{eff}}/\alpha_0 = 1$ are referred to as load attenuating and amplifying designs, respectively. As a general trend, Figs. 2–5 reveal that, similar to the case of swept metallic wings,[32] the swept-back and swept-forward composite wings yield the lift attenuation and amplification, respectively.

However, to get an idea of the strength of the tailoring technique in obtaining an improved aeroelastic design, it should be remarked that the six structural variants described earlier are characterized by an equal number of layers of similar ply orientations. The only element differentiating them is the relative position and sequence, across the thickness, of the laminae whose ply orientations are ± 45 deg.

In spite of this change, these wing configurations result in large variabilities of the lift amplification and attenuation. In this connection, Fig. 5, associated with wing structure d, reveals that for this case larger lift amplifications are exhibited as compared to the ones resulting within structures a, b, and c (see Figs. 2–4, respectively). Concerning the lift attenuation (occurring in the case of swept-back wings), an opposite trend is emerging. In this sense, the same figures reveal that structure d, resulting in the largest lift amplification as compared to the other ones, produces at the same time the largest relative lift attenuation.

Another conclusion arising from the comparison of Figs. 2–5 concerns the effect played by the warping restraint. From the case of metallic wings, it is known[17] that the warping restraint results in an increase of their torsional rigidity, its effect decaying with the increase of the wing aspect ratio. As a result, this effect will produce, from the static aeroelastic point of view, an attenuation of the lift amplification and, consequently, an increase of the divergence speed. However, as it was revealed previously in Ref. 15 and more thoroughly in this paper, in the case of a composite cantilevered wing structure, the warping inhibition could play a more complex role in the sense that it could result in 1) a small modification of the lift amplification and attenuation with respect to its free warping counterpart (see Figs. 2–4), or 2) a big contribution allowing one to attentuate the lift amplification when compared to its free warping counterpart (see Figs. 5 and 7).

Moreover, Fig. 4 reveals that the warping inhibition could yield an exacerbation of the lift amplification for both aft- and back-swept wings and even in the case of moderate-to-high aspect ratio wings. Figures 6–8 reveal that changing only the location of the sequence of the laminae characterized by the ± 45-deg angles could result in dramatic changes of the trend of the variation of q_D^{WR}/q_D^{FW} vs Æ. Although structures b, d, and f result in a trend like the one exhibited by the metallic-type cantilevered wings where the warping inhibition plays a beneficial stabilizing role (at least for small-to-moderate aspect ratio wings), in the case of structures a, c, and d (see the bottom half of Figs. 6–8), the warping inhibition plays an opposite, destabilizing role. In this case, its detrimental effect is more prominent in the case of moderate-to-high aspect ratio wings than in the case of their small aspect ratio counterpart. This effect, due to the elastic couplings occurring in the composite cantilevered wing, is similar (on another plane) to the aeroelastic bending-torsional coupling arising in the case of swept-forward wings and yielding the wash-in effect.

However, the ab initio prediction of the character (beneficial or detrimental) of the warping inhibition effect as a function of the given composite cantilevered structure constitutes, nevertheless, a task that, in spite of its importance, could not be clarified within this study.

As a successful FSW design depends on the ability to tailor the wing, the result is that a judicious design should search for a composite structure whose warping inhibition would result in a great amount of lift attenuation with respect to its free warping counterpart. Needless to say, such a design would result in a wing for which the warping inhibition would play a beneficial role from the static aeroelastic point of view.

If one keeps in mind that, in the case of a cantilevered wing, the warping restraint effect is inherently present and that the composite wing could be tailored, the goals would be not only to eliminate, in the case of a forward-swept wing, the wash-in effect, but also to use properly the warping inhibition effect.

Last, but not least, Figs. 2–8 constitute a convincing demonstration of the strength of the tailoring technique, allowing one to modify the trend of variation of the static aeroelastic characteristics.

Appendix: Expressions of Stiffness Quantities Entering the Aeroelastic Governing Equations

As shown in Ref. 6, the coupling stiffness parameters are expressed as

$$\tilde{D}_{22} \equiv c(D_{22} - B_{22}^2/A_{22})$$

$$\tilde{D}_{26} \equiv 2c(D_{26} - B_{22}B_{26}/A_{22})$$

$$\tilde{D}_{66} \equiv 4c(D_{66} - B_{26}^2/A_{22})$$

In these expressions, A_{ij}, B_{ij}, and D_{ij} denote the stretching, bending-stretching coupling, and bending stiffness quantities, respectively (see Ref. 36).

References

[1]Librescu, L., *Elastostatics and Kinetics of Anisotropic and Heterogeneous Sheel-Type Structures,* Noorhoff International, Leyden, The Netherlands, 1975, pp. 1–281.

[2]Hertz, T. J., Shirk, M. H., Ricketts, R. H., and Weisshaar, T. A., "On the Track of Practical Forward-Swept Wings," *Astronautics and Aeronautics*, Vol. 20, Jan. 1982, pp. 40–52.

[3]Weisshaar, T. A., "Tailoring for Aeroelastic Stability and Lateral Control Enhancement," *Second International Symposium on Aeroelasticity and Structural Dynamics*, DGLR-Bericht 85-02, Aachen, FRG, April 1985, pp. 415–425.

[4]Shirk, M. H., Hertz, T. J., and Weisshaar, T. A., "Aeroelastic Tailoring—Theory, Practice and Promise," *Journal of Aircraft*, Vol. 23, No. 1, 1986, pp. 6–18.

[5]Weisshaar, T. A., "Aeroelastic Tailoring—Creative Use of Unusual Materials," AIAA Paper 87-0976, April 1987.

[6]Crawley, E.F., and Dugundji, J., "Frequency Determination and Non-Dimensionalization for Composite Cantilever Plates," *Journal of Sound and Vibration*, Vol. 72, No. 1, 1980, pp. 1–10.

[7]Jensen, D. W., Crawley, E. F., and Dugundji, J., "Vibration of Cantilevered Graphite/Epoxy Plates with Bending-Torsion Coupling, *Journal of Reinforced Plastics and Composites*," Vol. 1, July 1982, pp. 254–269.

[8]Bauchau, O. A., Coffenberry, B. S., and Rehfield, L. W., "Composite Box Beam Analysis: Theory and Experiments," *Journal of Reinforced Plastics and Composites*, Vol. 6, Jan. 1987, pp. 25–34.

[9]Rehfield, L. W., Hodges, D. H., and Atilgan, A. R., "Some Considerations on the Nonclassical Behavior of Thin-Walled Composite Beams," presented at the American Helicopter Society National Technical Specialists' Meeting on Advanced Rotocraft Structures, Williamsburg, VA, Oct. 1988.

[10]Oybo, G. A., "Some Implications of Warping Restraint on the Behavior of Composite Anisotropic Beams," *Journal Aircraft*, Vol. 26, No. 2, 1989, pp. 187–189.

[11]Lottati, I., "Flutter and Divergence Aeroelastic Characteristics for Composite Forward Swept Cantilevered Wing," *Journal Aircraft*, Vol. 22, No. 11, 1985, pp. 1001–1007.

[12]Librescu, L., and Simovich, J., "A General Formulation for the Aeroelastic Divergence of Composite Swept Forward Wing Structures," *Journal of Aircraft*, Vol. 25, No. 4, 1988, pp. 364–371.

[13]Librescu, L., and Khdeir, A. A., "Aeroelastic Divergence of Sweptforward Composite Wings Including Warping Restraint Effect," *AIAA Journal*, Vol. 26, No. 11, 1988, pp. 1373–1377.

[14]Librescu, L., and Thangjitham, S., "The Influence of the Warping Restraint on the Spanwise Lift Distribution of Swept Forward Composite Wing Structures," AIAA Paper 88-2209, April 1988.

[15]Librescu, L., and Thangjitham, S., "The Static Aeroelastic Behavior of Sweptforward Composite Wing Structures Taking into Account Their Warping Restraint Effect," *Proceedings of the Fourth Japan-U.S. Conference on Composite Materials*, Technomic, 1988, pp. 914–922.

[16]Librescu, L., and Thangjitham, S., "The Warping Restraint Effect in the Critical and Subcritical Static Aeroelastic Behavior of Swept Forward Composite Wing Structures," Society of Automotive Engineers, Paper 891056, April 1989.

[17]Petre, A., Stanescu, C., and Librescu, L., "Aeroelastic Divergence of Multicell Wings Taking Their Fixing Restraints into Account," *Revue de Mecanique Appliquee*, Vol. 19, No. 6, 1961, pp. 689–698.

[18]Petre, A., *Teoria Aeroelasticitatii, Statiaca*, Edit. Academiei Republicii Socialiste Romania, Bucarest, 1966.

[19]Diederich, F. W., and Budiansky, B., "Divergence of Swept Wings," NACA TN-1680, 1948.

[20]Weisshaar, T. A., "Forward Swept Wings: Static Aeroelasticity," *Air Force Dynamics Laboratory*, AFFDL–TR-3087, June 1979.

[21]Weisshaar, T. A., "Divergence of Forward Swept Composite Wings," *Journal of Aircraft*, Vol. 17, No. 6, 1980, pp. 442–448.

[22]Rodden, W. P., "Aeroelastic Divergence of Unrestrained Vehicles," *Journal of Aircraft*, Vol. 18, No. 12, 1981, pp. 1072–1073.

[23]Rodden, W. P., "Aeroelastic Divergence of Unrestrained Vehicles," *Journal of Aircraft*, Vol. 21, No. 1, 1984, pp. 94–96.

[24]Rodden, W. P., and Bellinger, E. D., "Unrestrained Aeroelastic Divergence in a Dynamic Stability Analysis," *Journal of Aircraft*, Vol. 19, No. 9, 1982, pp. 796,797.

[25]Noll, T. L., Eastep, F. E., and Calico, R. A., "Active Suppression of Aeroelastic Instabilities on a Forward-Swept Wing," *Journal of Aircraft*, Vol. 21, No. 3, 1984, pp. 202–208.

[26]Dugundji, J., "Experimental Studies in Aeroelasticity of Unswept and Forward Swept Graphite/Epoxy Wings," *Recent Trends in Aeroelasticity, Structures and Structural Design*, edited by P. Hajela, Univ. Press of Florida, Gainesville, FL, 1987, pp. 149–160.

[27]Chen, G. S., and Dugundji, J., "Experimental Aeroelastic Behavior of Forward-Swept Graphite/Epoxy Wings with Rigid-Body Freedom," *Journal of Aircraft*, Vol. 24, No. 7, 1987, pp. 454–462.

[28]Chen, G. S., and Dugundji, J., "Aeroelastic Behavior of Forward Swept Graphite/Epoxy Wing Aircraft with Rigid Body Freedoms," Massachusetts Inst. of Techology, Cambridge, MA, TELAC Rept. 86-14, May 1986.

[29]Niblett, L. T., "Aeroelastic Divergence of Trimmed Aircraft," *Journal of Aircraft*, Vol. 23, No. 9, 1986, pp. 727–729.

[30]Lottati, I., "Aeroelastic Stability Characteristics of a Composite Swept Wing with Tip Weights for an Unrestrained Vehicle," *Journal of Aircraft*, Vol. 24, No. 11, 1987, pp. 793–802.

[31]Housner, J. M., and Stein, M., "Flutter Analysis of Swept-Wing Subsonic Aircraft with Parameter Studies of Composite Wings," NASA TN-07539, Sept. 1974.

[32]Oyibo, G. A., "Generic Approach to Determine Optimum Aeroelastic Characteristics for Composite Forward-Swept-Wing Aircraft," *AIAA Journal*, Vol. 22, No. 1, 1984, pp. 117–123.

[33]Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., *Aeroelasticity*, Adison-Wesley, Reading, MA, 1955.

[34]Lehman, L. L., "Hybrid State Vector Methods for Structural Dynamics and Aeroelastic Boundary Value Problems," Ph.D. Dissertation, Stanford Univ., Stanford, CA, Feb. 1982, also NASA CR Rept. 3591.

[35]Foersching, H. (ed.), *Static Aeroelastic Effects on High Performance Aircraft*, AGARD CP 403, July 1987.

[36]Jones, R. M., *Mechanics of Composite Materials*, McGraw-Hill, New York, 1975.